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ALGORITHM FOR COMPUTING SATELLITE ORIENTATION.
VALUES OF THE GRAVITATIONAL CONSTANTS

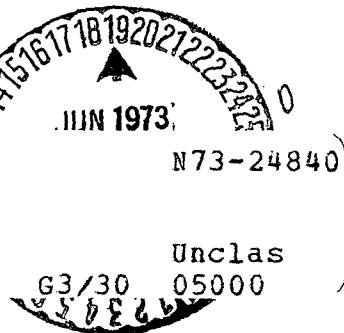
P. E. El'yasberg, V. V. Kugaenko,
V. M. Sinitsyn, V. E. Sokolov

Translation of "Algoritm rascheta orientatsii sputnika.
Znacheniya gravitatsionnykh postoyannykh".
Academy of Sciences, USSR. Institute for Space Research,
Report Rr-118, Moscow, 1972, 11pp.

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ALGORITHM FOR COMPUTING SATELLITE ORIENTATION.
VALUES OF THE GRAVITATIONAL CONSTANTS

P. E. El'yasberg, B. V. Kugaenko, V. M. Sinitsyn, V. E. Sokolov

I. Preliminary remarks.

Consider the following coordinate systems:

1. An absolute system of coordinates with axis \bar{I} directed toward the first point of Aries, the axis \bar{k} along the axis of rotation of the earth and the axis \bar{J} to complete a right-hand system.

The origin coincides with the center of the earth; in all of the following coordinate systems the origin is at the center of mass of the satellite.

2. A Koenig system of coordinates, with axes \bar{I} , \bar{K} and \bar{J} ; these axes parallel the axes of the absolute system.

3. An associated system of coordinates with axes $\bar{\zeta}_1$, $\bar{\zeta}_2$ and $\bar{\zeta}_3$ directed along the principal central axes of inertia of the satellite.

The designation of the axes of the associated system may be changed as a function of the mode of rotation of the satellite; this must be established in calculations in accordance with the stated algorithm.

4. A coordinate system tied to the kinetic moment vector \bar{L} of the satellite's rotation and to the Koenig system of coordinates such that the \bar{OL} -axis is directed along \bar{L} , the \bar{OL}_1 -axis lies in the \bar{OLK} plane, orthogonal to \bar{OL} , forming an acute angle with \bar{OK} ; the \bar{OL}_2 -axis completes the right-hand system.

5. A structural system of coordinates. This system is tied to the satellite. The matrix for converting from system 3 to system 5 is symbolized as F . The F -matrix is constant for a given satellite.

Calculation of satellite orientation reduces to determining the transformation matrix for conversion from system 2 to system 5.

This matrix is denoted by D. The transformation matrix /3 for converting from system 2 to system 3 is designated A.

If the D-matrix is known, any vector \bar{W}_{k_2} specified in system 2 will have the form in system 5

$$\bar{W}_{k_5} = D \bar{W}_{k_2} = FA \bar{W}_{k_2}.$$

The D-matrix is orthogonal, i.e. we have the relationship

$$\bar{W}_{k_2} = D^{-1} \bar{W}_{k_5} = D^T \bar{W}_{k_5}.$$

The matrix D^T is obtained from D by transposition. Since the F-matrix is constant for a given object it is convenient to consider only the calculation of the A-matrix.

Let the known matrices be:

B - transformation matrix from system 4 to system 3;
C - transformation matrix from system 2 to system 4..

Then $A = B \cdot C$.

The matrix B is known if the Euler angles fixing the position of system 3 relative to system 4 are determined:

φ - angle of proper rotation
 ψ - precession angle
 ϑ - nutation angle (see Fig. 1).

The matrix C can be determined if we know only ρ and σ , where ρ is the angle between the kinetic moment vector \bar{L} and the K-axis of the Koenig system, and σ is the longitude of the vector \bar{L} in the Koenig system of coordinates (see Fig. 2). /4

II. Initial Data.

The initial data for calculating the orientation are the quantities: $f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, \omega, \rho, \sigma$, -- four valued numbers (units, tenths, hundredths and thousandths of radians or rad/sec, corresponding to their physical sense in formulas (2), (3) and (4)), f_1, f_5 - five-valued numbers (units, tenths, hundredths, thousandths and ten-thousandths of radians/sec). In addition, assigned are: t_0 - the initial time interval (hours, minutes) and t_k - the final time interval (hours, minutes).

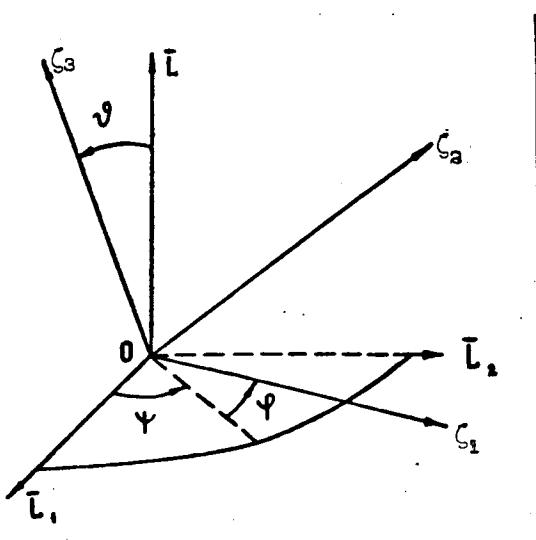
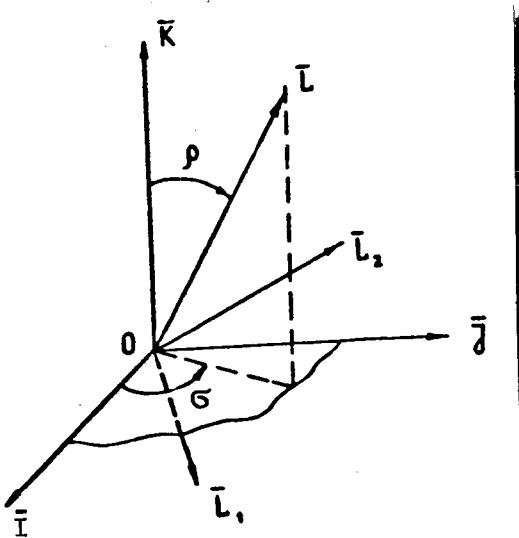


Fig. 1.



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Fig. 2.

III. Computing Algorithm.

Calculation of the orientation is carried out in the following sequence:

(1) $\Delta t = t - t_o$, where t is the instant of time pertaining to $[t_o, t_k]$ in which the orientation is determined. Δt is established in seconds.

$$(2) \quad \psi + \varphi = f_0 + 0.1f_1 \Delta t + f_2 \sin(0.01\omega \Delta t) + f_3 \cos(0.01\omega \Delta t);$$

$$(3) \quad \psi - \varphi = f_4 + 0.1f_5 \Delta t + f_6 \sin(0.01\omega \Delta t) + f_7 \cos(0.01\omega \Delta t);$$

$$(4) \quad \theta = f_8 + f_9 \sin(0.01\omega \Delta t) + f_{10} \cos(0.01\omega \Delta t);$$

$$(5) \quad \lambda_0 = \cos(\vartheta/2) \cdot \cos[(\psi + \varphi)/2];$$

$$(6) \quad \lambda_1 = \sin(\vartheta/2) \cdot \cos[(\psi - \varphi)/2];$$

$$(7) \quad \lambda_2 = \sin(\vartheta/2) \cdot \sin[(\psi - \varphi)/2];$$

$$(8) \quad \lambda_3 = \cos(\vartheta/2) \cdot \sin[(\psi + \varphi)/2];$$

$$(9) \quad B = \begin{vmatrix} \lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2 & 2(\lambda_0\lambda_3 + \lambda_1\lambda_2) & 2(\lambda_1\lambda_3 - \lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2 - \lambda_0\lambda_3) & \lambda_0^2 + \lambda_2^2 - \lambda_1^2 - \lambda_3^2 & 2(\lambda_0\lambda_1 + \lambda_2\lambda_3) \\ 2(\lambda_0\lambda_4 + \lambda_1\lambda_3) & 2(\lambda_2\lambda_3 - \lambda_0\lambda_1) & \lambda_0^2 + \lambda_3^2 - \lambda_1^2 - \lambda_2^2 \end{vmatrix};$$

$$(10) \quad C = \begin{vmatrix} \cos\sigma \cdot \cos\beta & \sin\sigma \cdot \cos\beta & -\sin\beta \\ -\sin\sigma & \cos\sigma & 0 \\ \cos\sigma \cdot \sin\beta & \sin\sigma \cdot \sin\beta & \cos\beta \end{vmatrix}$$

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Remarks. The matrix C is constant over the time interval $t_0 \leq t \leq t_k$.

$$(11) \quad A = B \cdot C$$

$$(12) \quad D = F \cdot A$$

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Values of the gravitational constants
for use in "Algorithms for computing
satellite orientation"

n	m	$d_{nm} [m^2/sec^2]$	$\beta_{nm} [m^2/sec^2]$	n	m	$d_{nm} [m^2/sec^2]$	$\beta_{nm} [m^2/sec^2]$
1	2	3	4	1	2	3	4
2	0	$-0.67886473 \cdot 10^5$		6	2	$0.39 \cdot 10^0$	$-0.274 \cdot 10^1$
2	1	0	6	6	3	$0.39 \cdot 10^{-1}$	$0.52 \cdot 10^{-1}$
2	2	$0.9755 \cdot 10^2$	$-0.5505 \cdot 10^2$	6	4	0	$-0.954 \cdot 10^{-1}$
3	0	$0.159 \cdot 10^3$		6	5	$-0.107 \cdot 10^{-1}$	$-0.264 \cdot 10^{-1}$
3	1	$0.1336 \cdot 10^3$	$0.176 \cdot 10^2$	6	6	$0.13 \cdot 10^{-2}$	$-0.10 \cdot 10^{-2}$
3	2	$0.191 \cdot 10^2$	$-0.135 \cdot 10^2$	7	0	$0.22 \cdot 10^2$	
3	3	$0.604 \cdot 10^1$	$0.1252 \cdot 10^2$	7	1	$0.111 \cdot 10^2$	$0.554 \cdot 10^1$
4	0	$0.999 \cdot 10^2$		7	2	$0.176 \cdot 10^1$	$0.100 \cdot 10^1$
4	1	$-0.316 \cdot 10^2$	$-0.292 \cdot 10^2$	7	3	$0.178 \cdot 10^0$	$-0.204 \cdot 10^0$
4	2	$0.464 \cdot 10^1$	$0.998 \cdot 10^1$	7	4	$-0.268 \cdot 10^{-1}$	$-0.147 \cdot 10^{-1}$
4	3	$0.372 \cdot 10^1$	$-0.563 \cdot 10^0$	7	5	0	$0.223 \cdot 10^{-2}$
4	4	$-0.11 \cdot 10^0$	$0.451 \cdot 10^0$	7	6	$-0.114 \cdot 10^{-2}$	$0.438 \cdot 10^{-3}$
5	0	$0.15 \cdot 10^2$		7	7	$0.187 \cdot 10^{-3}$	$-0.82 \cdot 10^{-4}$
5	1	$-0.27 \cdot 10^1$	$-0.539 \cdot 10^1$	8	0	$0.78 \cdot 10^1$	
5	2	$0.621 \cdot 10^1$	$-0.356 \cdot 10^1$	8	1	$0.13 \cdot 10^1$	$0.13 \cdot 10^1$
5	3	$-0.894 \cdot 10^0$	$-0.19 \cdot 10^0$	8	2	$0.26 \cdot 10^0$	$0.41 \cdot 10^0$
5	4	$-0.132 \cdot 10^0$	$0.39 \cdot 10^{-1}$	8	3	$-0.38 \cdot 10^{-1}$	$0.13 \cdot 10^{-1}$
5	5	$0.201 \cdot 10^{-1}$	$-0.930 \cdot 10^{-1}$	8	4	$-0.124 \cdot 10^{-1}$	$0.66 \cdot 10^{-2}$
6	0	$-0.318 \cdot 10^2$		8	5	$-0.69 \cdot 10^{-3}$	$0.69 \cdot 10^{-3}$
6	1	$-0.496 \cdot 10^1$	$0.20 \cdot 10^1$	8	6	$-0.88 \cdot 10^{-4}$	$0.458 \cdot 10^{-3}$

1	2	3	4	1	2	3	4
8	7	$0.97 \cdot 10^{-5}$	$0.29 \cdot 10^{-4}$	11	3	$-0.18 \cdot 10^{-1}$	$-0.38 \cdot 10^{-1}$
8	8	$-0.64 \cdot 10^{-5}$	$0.56 \cdot 10^{-5}$	11	4	$-0.80 \cdot 10^{-3}$	$0.13 \cdot 10^{-2}$
9	0	$0.55 \cdot 10^1$		11	5	$0.76 \cdot 10^{-4}$	$0.328 \cdot 10^{-3}$
9	1	$0.575 \cdot 10^1$	$-0.82 \cdot 10^0$	11	6	$0.10 \cdot 10^{-4}$	$0.25 \cdot 10^{-5}$
9	2	$0.44 \cdot 10^{-1}$	$-0.35 \cdot 10^0$	11	7	$0.13 \cdot 10^{-5}$	$-0.368 \cdot 10^{-5}$
9	3	$-0.478 \cdot 10^{-1}$	$-0.573 \cdot 10^{-1}$	11	8	$0.18 \cdot 10^{-6}$	$-0.60 \cdot 10^{-7}$
9	4	$0.32 \cdot 10^{-2}$	$0.595 \cdot 10^{-2}$	11	9	$0.467 \cdot 10^{-7}$	$-0.39 \cdot 10^{-8}$
9	5	$-0.65 \cdot 10^{-4}$	0	11	10	$-0.721 \cdot 10^{-8}$	$-0.12 \cdot 10^{-8}$
9	6	$0.17 \cdot 10^{-4}$	$0.184 \cdot 10^{-3}$	11	11	$0.154 \cdot 10^{-8}$	$-0.51 \cdot 10^{-9}$
9	7	$-0.60 \cdot 10^{-5}$	$-0.157 \cdot 10^{-4}$	12	0	$0.32 \cdot 10^1$	
9	8	$0.475 \cdot 10^{-5}$	$0.12 \cdot 10^{-5}$	12	1	$-0.18 \cdot 10^1$	$-0.11 \cdot 10^1$
9	9	$-0.39 \cdot 10^{-6}$	$0.44 \cdot 10^{-6}$	12	2	$0.87 \cdot 10^{-1}$	$0.23 \cdot 10^0$
10	0	$0.23 \cdot 10^2$		12	3	$0.14 \cdot 10^{-1}$	$0.12 \cdot 10^{-1}$
10	1	$0.430 \cdot 10^1$	$-0.391 \cdot 10^1$	12	4	$-0.79 \cdot 10^{-3}$	$-0.39 \cdot 10^{-3}$
10	2	$-0.11 \cdot 10^0$	$-0.376 \cdot 10^0$	12	5	$0.34 \cdot 10^{-4}$	$0.68 \cdot 10^{-4}$
10	3	$-0.74 \cdot 10^{-2}$	$-0.516 \cdot 10^{-1}$	12	6	$-0.30 \cdot 10^{-5}$	0
10	4	$-0.19 \cdot 10^{-2}$	$-0.150 \cdot 10^{-2}$	12	7	$0.14 \cdot 10^{-6}$	$0.141 \cdot 10^{-5}$
10	5	$-0.31 \cdot 10^{-3}$	$-0.550 \cdot 10^{-3}$	12	8	$-0.14 \cdot 10^{-7}$	$0.42 \cdot 10^{-7}$
10	6	$-0.13 \cdot 10^{-4}$	$-0.878 \cdot 10^{-4}$	12	9	$-0.46 \cdot 10^{-8}$	$0.61 \cdot 10^{-8}$
10	7	$0.32 \cdot 10^{-5}$	$0.16 \cdot 10^{-5}$	12	10	$-0.39 \cdot 10^{-9}$	0
10	8	$0.51 \cdot 10^{-6}$	$-0.58 \cdot 10^{-6}$	12	11	$-0.11 \cdot 10^{-9}$	$-0.14 \cdot 10^{-9}$
10	9	$-0.12 \cdot 10^{-7}$	$0.12 \cdot 10^{-7}$	12	12	$-0.11 \cdot 10^{-10}$	$-0.34 \cdot 10^{-10}$
10	10	$0.315 \cdot 10^{-7}$	$-0.79 \cdot 10^{-8}$	13	0	$0.66 \cdot 10^1$	
11	0	$-0.12 \cdot 10^2$		13	1	$-0.21 \cdot 10^1$	$0.10 \cdot 10^1$
11	1	0	$0.11 \cdot 10^1$	13	2	$-0.13 \cdot 10^0$	$0.52 \cdot 10^{-1}$
11	2	$0.16 \cdot 10^0$	$-0.30 \cdot 10^0$	13	3	$0.39 \cdot 10^{-2}$	$-0.58 \cdot 10^{-2}$

1	2	3	4	1	2	3	4
13	4	$-0.30 \cdot 10^{-3}$	$0.89 \cdot 10^{-3}$	15	2	$-0.84 \cdot 10^{-1}$	$-0.42 \cdot 10^{-1}$
13	5	$0.117 \cdot 10^{-3}$	$-0.59 \cdot 10^{-4}$	15	3	$0.14 \cdot 10^{-2}$	$-0.14 \cdot 10^{-2}$
13	6	$-0.76 \cdot 10^{-5}$	$0.57 \cdot 10^{-5}$	15	4	$0.91 \cdot 10^{-4}$	$0.64 \cdot 10^{-3}$
13	7	$-0.40 \cdot 10^{-6}$	0	15	5	$0.18 \cdot 10^{-4}$	0
13	8	$-0.29 \cdot 10^{-7}$	$-0.14 \cdot 10^{-7}$	15	6	$0.30 \cdot 10^{-5}$	$-0.507 \cdot 10^{-5}$
13	9	$-0.20 \cdot 10^{-8}$	$0.750 \cdot 10^{-8}$	15	7	$0.360 \cdot 10^{-6}$	$0.12 \cdot 10^{-6}$
13	10	$-0.64 \cdot 10^{-9}$	$-0.71 \cdot 10^{-10}$	15	8	$-0.221 \cdot 10^{-7}$	$-0.88 \cdot 10^{-8}$
13	11	$-0.25 \cdot 10^{-10}$	$0.50 \cdot 10^{-10}$	15	9	$0.34 \cdot 10^{-9}$	$0.51 \cdot 10^{-9}$
13	12	0	$0.95 \cdot 10^{-11}$	15	10	$-0.20 \cdot 10^{-10}$	0
13	13	$-0.16 \cdot 10^{-11}$	$0.16 \cdot 10^{-11}$	15	11	$-0.37 \cdot 10^{-11}$	$0.11 \cdot 10^{-10}$
14	0	$0.34 \cdot 10^1$		15	12	$0.12 \cdot 10^{-12}$	$0.12 \cdot 10^{-12}$
14	1	$-0.67 \cdot 10^0$	$0.17 \cdot 10^1$	15	13	$-0.51 \cdot 10^{-13}$	0
14	2	$0.69 \cdot 10^{-1}$	$-0.12 \cdot 10^0$	15	14	$0.17 \cdot 10^{-14}$	$-0.51 \cdot 10^{-14}$
14	3	$0.32 \cdot 10^{-2}$	0	15	15	0	$-0.31 \cdot 10^{-15}$
14	4	$0.12 \cdot 10^{-3}$	$-0.46 \cdot 10^{-3}$	16	0	$-0.11 \cdot 10^2$	
14	5	$-0.25 \cdot 10^{-4}$	$-0.17 \cdot 10^{-4}$	16	1	$-0.63 \cdot 10^0$	$0.25 \cdot 10^1$
14	6	$0.12 \cdot 10^{-5}$	$-0.37 \cdot 10^{-5}$	16	2	$0.38 \cdot 10^{-1}$	$0.57 \cdot 10^{-1}$
14	7	$0.48 \cdot 10^{-7}$	$0.48 \cdot 10^{-7}$	16	3	$-0.59 \cdot 10^{-2}$	$0.35 \cdot 10^{-2}$
14	8	$-0.12 \cdot 10^{-7}$	$-0.23 \cdot 10^{-7}$	16	4	$-0.73 \cdot 10^{-4}$	$0.29 \cdot 10^{-3}$
14	9	$0.16 \cdot 10^{-8}$	$0.30 \cdot 10^{-8}$	16	5	$-0.18 \cdot 10^{-4}$	$0.14 \cdot 10^{-4}$
14	10	$0.15 \cdot 10^{-9}$	$-0.12 \cdot 10^{-9}$	16	6	$-0.18 \cdot 10^{-5}$	$-0.12 \cdot 10^{-5}$
14	11	$0.90 \cdot 10^{-11}$	$-0.24 \cdot 10^{-10}$	16	7	$0.213 \cdot 10^{-6}$	$0.19 \cdot 10^{-7}$
14	12	$0.34 \cdot 10^{-12}$	$-0.20 \cdot 10^{-11}$	16	8	$-0.11 \cdot 10^{-7}$	0
14	13	$0.23 \cdot 10^{-12}$	$0.23 \cdot 10^{-12}$	16	9	$0.93 \cdot 10^{-10}$	$-0.103 \cdot 10^{-8}$
14	14	$-0.44 \cdot 10^{-13}$	$-0.88 \cdot 10^{-14}$	16	10	$-0.21 \cdot 10^{-10}$	0
15	0	$0.11 \cdot 10^2$		16	11	$0.54 \cdot 10^{-12}$	$-0.38 \cdot 10^{-11}$
15	1	0	$0.13 \cdot 10^1$	16	12	$0.92 \cdot 10^{-13}$	$-0.14 \cdot 10^{-12}$

1	2	3	4	1	2	3	4
16	13	$0.17 \cdot 10^{-13}$	$0.85 \cdot 10^{-14}$	19	0	$0.12 \cdot 10^2$	
16	14	$-0.45 \cdot 10^{-15}$	$-0.90 \cdot 10^{-15}$	19	12	$0.31 \cdot 10^{-1}$	$-0.44 \cdot 10^{-14}$
16	15	$-0.23 \cdot 10^{-15}$	0	19	13	$0.89 \cdot 10^{-15}$	$-0.18 \cdot 10^{-14}$
16	16	$-0.30 \cdot 10^{-16}$	$0.10 \cdot 10^{-16}$	19	14	0	$-0.42 \cdot 10^{-6}$
17	0	$-0.38 \cdot 10^1$		20	0	0	
17	12	$0.16 \cdot 10^{-12}$	0	20	13	$0.84 \cdot 10^{-15}$	$0.42 \cdot 10^{-15}$
17	13	$0.48 \cdot 10^{-14}$	0	20	14	$0.90 \cdot 10^{-17}$	$-0.19 \cdot 10^{-16}$
17	14	$-0.29 \cdot 10^{-15}$	$0.43 \cdot 10^{-15}$	21	0	$-0.84 \cdot 10^1$	
18	0	$0.16 \cdot 10^2$		21	13	0	$-0.14 \cdot 10^{-15}$
18	12	$0.90 \cdot 10^{-14}$	$0.90 \cdot 10^{-14}$	21	14	$0.21 \cdot 10^{-1}$	0
18	13	0	$-0.27 \cdot 10^{-14}$	22	14	$-0.20 \cdot 10^{-17}$	$0.60 \cdot 10^{-17}$
18	14	$-0.16 \cdot 10^{-15}$	$-0.26 \cdot 10^{-15}$				

$$a_0 = 6378140 \text{ m};$$

$$\mu = 0.3986013 \cdot 10^{15} \text{ m}^3/\text{sec}^2;$$

$$\bar{\lambda} = 1/298.25;$$

$$\omega_3 = 0.729211575 \cdot 10^{-4} \text{ 1/sec};$$

$$R = 6371000 \text{ m};$$

$$1 \text{ A.U.} = 1.496 \cdot 10^{11} \text{ m};$$

$$\mu_0 = 0.132712517 \cdot 10^{21} \text{ m}^3/\text{sec}^2;$$

$$\mu_1 = 0.49027779 \cdot 10^{13} \text{ m}^3/\text{sec}^2.$$